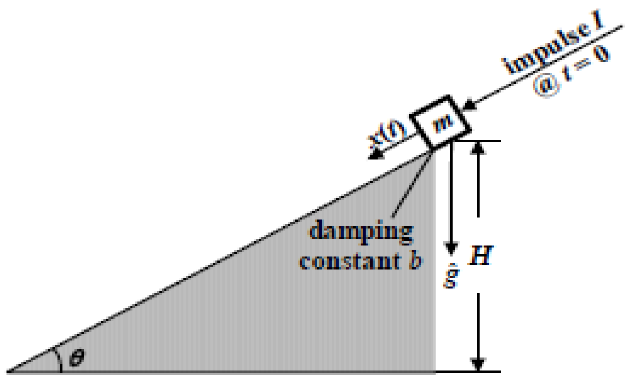
CITY COLLEGE

CITY UNIVERSITY OF NEW YORK

HOMEWORK #1



MASS SLIDING ON THE VICIOUS PLANE

ME 411: System Modeling Analysis and Control

Fall 2010

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Abstract

In system dynamics, mathematical modeling of dynamic systems is an essential tool for the study of the physics behind the control system. For our study, a small mass slide on the incline viscous plane forced with an impulse from its initial condition of zero velocity and zero position. Analytical method was used for solving ODEs and for numerical calculations graphical and symbolic mathematical software such as Maple and MatLab was used. Euler method, trapezoidal and bisection method were used for numerical calculation. Potential energy, kinetic energy and total heat dissipated were also evaluated for the give condition as per the assigned parameters.

1.0 Nomenclature

$F = force\ (N)$

$m = mass\ (kg)$

$b = viscous\ \left(\frac{N-s}{m}\right)$

$g = gravity\ \left(\frac{m}{s^2}\right)$

$H = height\ (m)$

$I = impulse\ (N-s)$

$v = velocity\ (\frac{m}{s})$

$x = position\ (m)$

$\theta = angle\ (deg)$

$t = time\ (s)$

$f_b = damping\ force\ (N)$

1. Background

A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. In very general sense, it can be represented as a functional relationship of the form

$$\boldsymbol{Dependent\ Variable} = \boldsymbol{f}\left(\begin{matrix} & \boldsymbol{independent\ variables,} \\ \boldsymbol{parameters,} & \boldsymbol{forcing\ fucntions} \end{matrix}\right)$$

where, the dependent variable is a characteristic that usually reflects the behavior or state of the system. Independent variables are usually dimensions, such as time and space, the parameters are reflective of the systems properties or composition and the forcing functions are external influences acting upon the system.

2. Theory

The mathematical expression, or model, of the second law is the famous Newton second law of motion, given by the equation,

$$F = ma - - - - - - - - - - (i)$$

where,

$F = force\ (N)$

$m = mass\ (kg)$

$a = acceleration\ \left(\frac{m}{s^2}\right)$

For our given system,

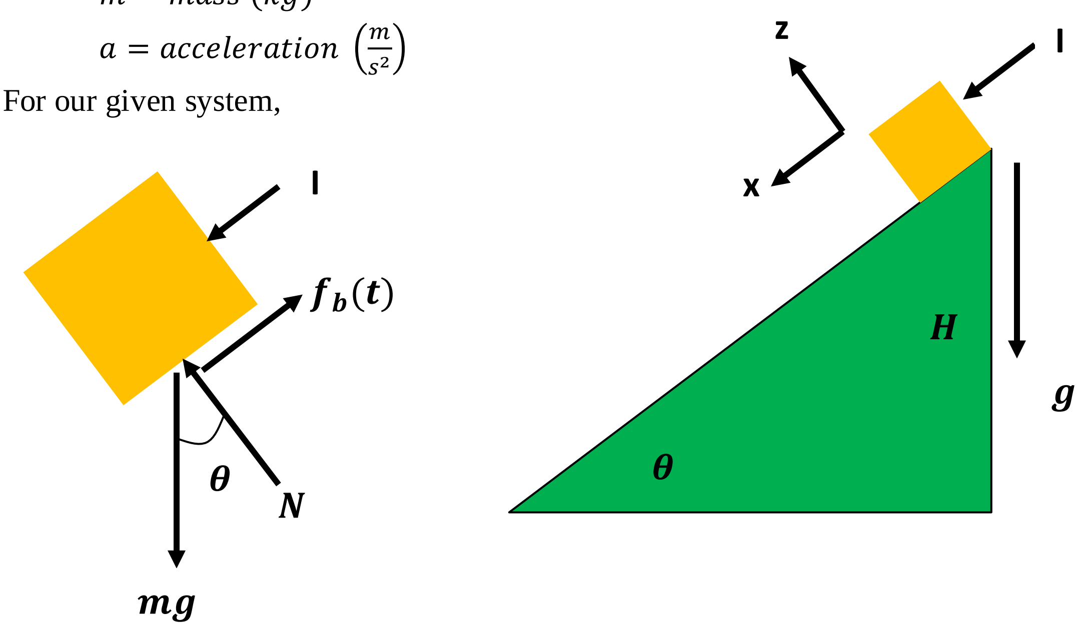


Fig.1.0 Free Body Diagram

In Z-direction,

$$\sum F_z = ma_z$$

$$N - mgcos\theta = 0 \qquad (* \ a_z = 0)$$
$$N = mgcos\theta --------- (ii)$$

In X-direction,

$$\sum F_x = ma_x$$

$$I * \delta t + mgsin\theta - f_b = ma_x \qquad (* \ f_b = bv(t))$$
$$\frac{I * \delta t}{m} + mgsin\theta - bv(t) = m\dot{v(t)}$$

For further simplification of the equation above, by Dirac delta function,

a) There required governing first order differential equation of motion is,

$$\dot{v(t)} + \frac{b}{m}v(t) = gsin\theta \ -------- (iii) \quad with \ initial \ Condition, \begin{bmatrix} x(o^+) = 0 \\ v(o^+) = \frac{I}{m} \end{bmatrix}$$

3. ODE method

We have the governing differential equation of motion

$$m\dot{v(t)} + bv(t) = gsin\theta$$

Part 1: Homogeneous solution

Write the characteristics solution.

$$r + \frac{b}{m} = 0 \rightarrow r = -\frac{b}{m}$$
$$v_h(t) = C_1 e^{\frac{-bt}{m}}$$

Part 2: Non -homogeneous solution

Assumed the particular solution for the equation is constant.

$$v_p(t) = c \ and \ \dot{v(t)} = 0$$
$$0 + \frac{b}{m}c = gsin\theta$$
$$c = \frac{mgsin\theta}{b}$$
$$v(t) = v_h(t) + v_p(t)$$
$$v(t) = C_1 e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$Using\ initial\ conditions\ @\ t = 0, V(0^+) = \frac{I}{m}$

$$v(0) = C_1 e^{\frac{-b*o}{m}} + \frac{mgsin\theta}{b}$$

$$C_1 = \frac{I}{m} - \frac{mgsin\theta}{b}$$

$Therefore, the\ equations\ for\ the\ velocity\ is,$

$$b)\ v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b} ----(iv)$$

$Again, integrating\ the\ equations\ ---\ (iv)$

$$\frac{dx}{dt} = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$x(t) = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + C$$

$Using\ initial\ conditions\ @\ t = 0, X(0^+) = 0$

$$x(t) = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + C$$

$$0 = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) + 0 + C$$

$$C = \left(\frac{I}{b} - \frac{m^2 gsin\theta}{b^2}\right)$$

$Therefore, the\ equations\ for\ the\ velocity\ is,$

$$b)\ x(t) = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + \left(\frac{I}{b} - \frac{m^2 gsin\theta}{b^2}\right) ----(v)$$

4. Laplace Transformation

$The\ equations\ for\ the\ velocity\ is$

$$\dot{v(t)} + \frac{b}{m}v(t) = gsin\theta$$

$The\ Laplace\ transformation\ of\ the\ equations, \mathcal{L}(v(t)) = V(s)$

$$sV(s) - v(0) + \frac{b}{m}V(s) = \frac{gsin\theta}{s}$$

$$V(s)\left(s + \frac{b}{m}\right) = \frac{gsin\theta}{s} + \frac{I}{m}$$

$$V(s)\left(\frac{ms+b}{\cancel{m}}\right) = \frac{mgsin\theta+Is}{\cancel{m}s}$$

$$V(s) = \frac{mgsin\theta+Is}{s(ms+b)}$$

$$V(s) = \frac{mgsin\theta}{s(ms+b)} + \frac{I\cancel{s}}{\cancel{s}(ms+b)}$$

$$V(s) = \frac{gsin\theta}{s(s+\frac{b}{m})} + \frac{I}{m(s+\frac{b}{m})}$$

$$\mathcal{L}^{-1}[V(s)] = gsin\theta\mathcal{L}^{-1}\left[\frac{1}{s(s+\frac{b}{m})}\right] + \frac{I}{m} * \mathcal{L}^{-1}\left[\frac{1}{(s+\frac{b}{m})}\right]$$

$$v(t) = gsin\theta * \frac{m}{b}\left[1 - e^{-\left(\frac{bt}{m}\right)}\right] + \frac{I}{m}e^{-\left(\frac{bt}{m}\right)}$$

$$v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$The\ equations\ for\ the\ position\ is$

$$\ddot{x(t)} + \frac{b}{m}\dot{x(t)} = gsin\theta$$

$The\ Laplace\ transformation\ of\ the\ equations, \mathcal{L}(x(t)) = X(s)$

$$s^2X(s) - sx(0) - \dot{x(0)} + \frac{b}{m}[sX(s) - x(0)] = \frac{gsin\theta}{s}$$

$$X(s)\left(s^2 + \frac{bs}{m}\right) = \frac{gsin\theta}{s} + \frac{I}{m}$$

$$X(s)\left(\frac{ms^2+bs}{\cancel{m}}\right) = \frac{mgsin\theta+Is}{\cancel{m}s}$$

$$X(s) = \frac{mgsin\theta+Is}{s(ms^2+bs)}$$

$$X(s) = \frac{mgsin\theta}{s(ms^2+bs)} + \frac{I\cancel{s}}{\cancel{s}(ms^2+bs)}$$

$$X(s) = \frac{gsin\theta}{s(s^2+\frac{bs}{m})} + \frac{I}{m(s^2+\frac{bs}{m})}$$

$$\mathcal{L}^{-1}[X(s)] = gsin\theta\mathcal{L}^{-1}\left[\frac{1}{s^2(s+\frac{b}{m})}\right] + \frac{I}{m} * \mathcal{L}^{-1}\left[\frac{1}{s(s+\frac{b}{m})}\right]$$

$$x(t) = gsin\theta * \frac{m^2}{b^2}\left[\frac{b}{m}t - 1 + e^{-\left(\frac{bt}{m}\right)}\right] + \frac{I}{m} * \frac{m}{b}\left(1 - e^{-\left(\frac{bt}{m}\right)}\right)$$

$$x(t) = \frac{mgsin\theta t}{m} - \frac{m^2gsin\theta}{b^2} + \frac{m^2gsin\theta}{b^2}e^{-\left(\frac{bt}{m}\right)} + \frac{I}{b} - \frac{I}{b}e^{-\left(\frac{bt}{m}\right)}$$

$$x(t) = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta*t}{b} + \left(\frac{I}{b} - \frac{m^2gsin\theta}{b^2}\right)$$

5. Time to reach the bottom.

Case 1: *Time to reach the bottom of the slide for viscid case*, $b \neq 0$,

$$\frac{H}{sin\theta} = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + \left(\frac{I}{b} - \frac{m^2 gsin\theta}{b^2}\right)$$

Time to reach the bottom of the slide for viscid case, b ≠ 0, and the transcendental equations was solved in symbolic mathematical software Maple 13. However, Impulse, I, is represented by, f, in the equation as, I, is an inbuilt parameter for imaginary number in Maple software.

$$solve\left(-\frac{m}{b}\left(\frac{f}{m} - \frac{m \cdot g \cdot \sin(\Theta)}{b}\right)\cdot \exp^{-\frac{b \cdot t_b}{m}} + \frac{m \cdot g \cdot \sin(\Theta) \cdot t_b}{b} - \frac{m^2 \cdot g \cdot \sin(\Theta)}{b^2} + \frac{f}{b} = \frac{H}{\sin(\Theta)}, t_b\right)$$

$$t_b =$$

$$\frac{1}{b \sin(\Theta)^2 g\, m \ln(\exp)}\Bigg(\ln(\exp) H b^2 - \ln(\exp) f b \sin(\Theta) + \sin(\Theta)^2 m^2 g \operatorname{LambertW}\left(\frac{1}{\sin(\Theta) m^2 g}\left(\ln(\exp)\left(f b - m^2 g \sin(\Theta)\right) e^{-\frac{\ln(\exp)\left(H b^2 + m^2 g \sin(\Theta)^2 - f b \sin(\Theta)\right)}{m^2 g \sin(\Theta)^2}}\right)\right) + \ln(\exp) m^2 g \sin(\Theta)^2\Bigg)$$

$$- - - - - - - (vi)$$

Case 2: *Time to reach the bottom of the slide for viscid case*, $b = 0$,

$$\dot{v}(t) + \frac{b}{m}v(t) = gsin\theta$$

$$\frac{d^2x}{dt^2} = gsin\theta$$

$$Intregrating\ the\ equation\ twice\ with\ respect\ to, t\ as\ a\ variable$$

$$v(t) = gsin\theta t + c_1 --------------------(v)$$

$$x(t) = \frac{1}{2} gsin\theta t^2 + c_1 t + c_2 ----------------(vi)$$

$$Using\ the\ boundary\ condition, x(0) = 0, v(0) = \frac{I}{m}$$

$$\frac{I}{m} = gsin\theta 0 + c_1 \to c_1 = \frac{I}{m}$$

$$0 = \frac{1}{2} gsin\theta 0^2 + c_1 0 + c_2 \to c_2 = 0$$

$$Using\ the\ constant, in\ eqaution ----(vi)$$

$$x(t) = \frac{1}{2} gsin\theta t^2 + \frac{I}{m} t --------------(vii)$$

Time to reach the bottom of the slide (t) = t_b, X (t) = H/sinθ

$$\frac{H}{\sin\theta} = \frac{1}{2} gsin\theta t_b^2 + \frac{I}{m} t_b$$

$$> solve\left(\frac{H}{\sin(\Theta)} = \frac{1}{2} \cdot g \cdot \sin(\Theta) \cdot t^2 + \frac{J}{m} \cdot t, t\right)$$

$$-\frac{J - \sqrt{J^2 + 2\, g\, m^2\, H}}{g\, m \sin(\Theta)}, -\frac{J + \sqrt{J^2 + 2\, g\, m^2\, H}}{g\, m \sin(\Theta)}$$ *Ignoring the negative root as time is always postive*

$$t_b = -\frac{I + \sqrt{I^2 + 2gm^2H}}{mgsin\theta} -------------(viii)$$

6. Terminal speed

$$Terminal\ velocity\ of\ the\ falling\ boject\ when\ the\ bock\ speed\ as\ time\ (t) \to \infty\ is$$

$$v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$v(\infty) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-b\infty}{m}} + \frac{mgsin\theta}{b} \to \frac{mgsin\theta}{b}$$

$$v_\infty = \frac{mgsin\theta}{b} -------------------(ix)$$

7. Characteristics roots, the corresponding constant , the residues

Assumed data for MatLab Numerical calculation:

m	b	g	H	θ	I
[kg]	**[N-s/m]**	**[m/s]**	**[m]**	**[deg]**	**[N-s]**
2	**5**	**9.8**	**10**	**30**	**1**

MatLab Results under given inputs:

MatLab Results under given inputs:			
Constant coefficient of homogenous solution	.5840	-.5840	
Residues	.5840	-.5840	1.96
Roots	0	-0.5	

```
Command Window
The characteristic roots for dx^2/dt^2+(b/m)*dx/dt=gsin(theta):
r1:
     0

r2:
   -2.5000

-------------------------------------------------------------------------
The constant of homogeneous solution for dx^2/dt^2+(b/m)*dx/dt=gsin(theta):
C1:
    0.5840

C2:
   -0.5840

-------------------------------------------------------------------------
The residues of given laplace transformation  X(s) =(mgsin(theta)+Is)/(ms^3+bs)) is
Residue_1:
    0.5840

Residue_2:
   -0.5840

Residue_3:
    1.9600

fx >>
```

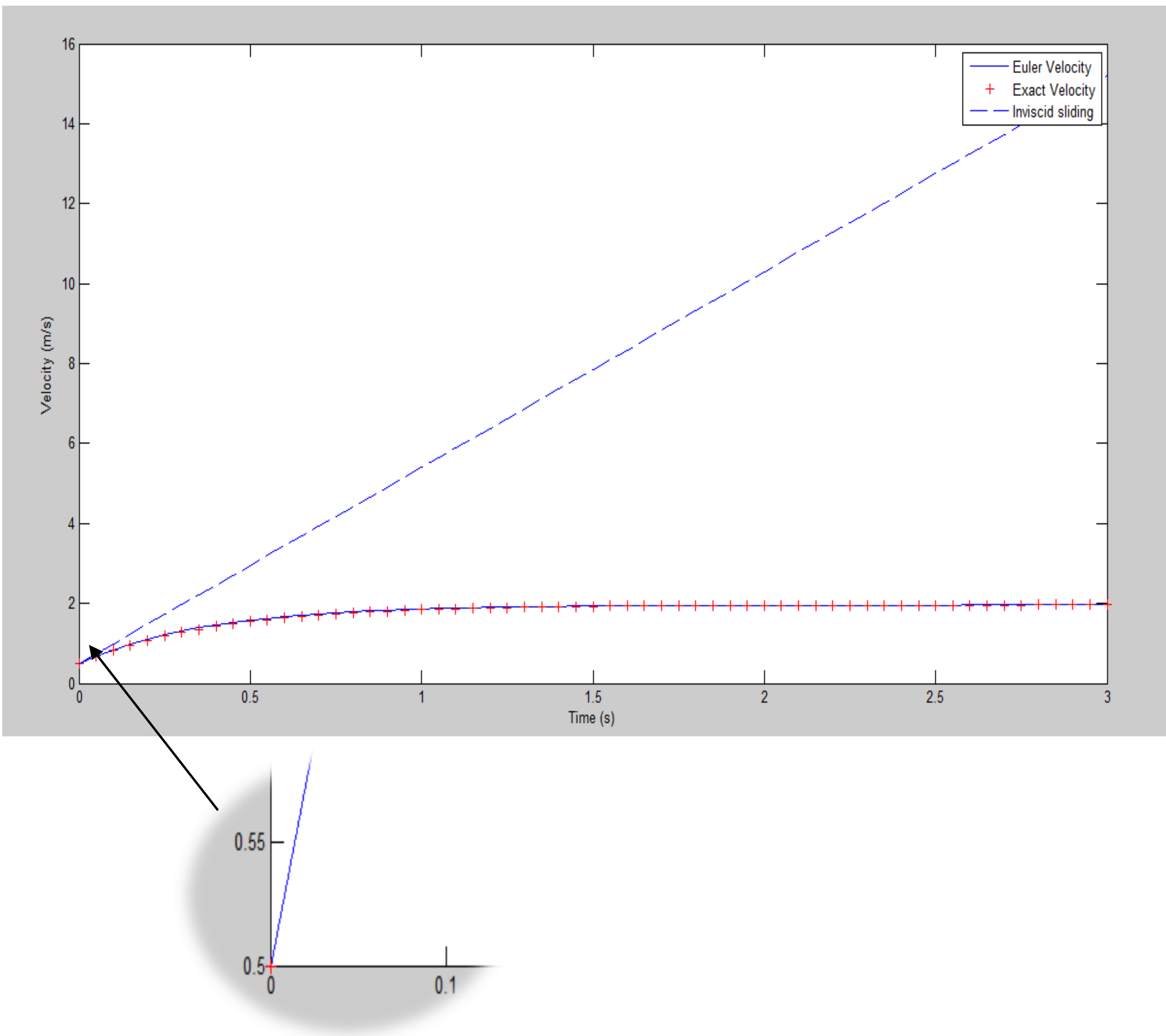
8. Numerical Solution of ODE

Euler's method for velocity:

$$\dot{v}(t) + \frac{b}{m}v(t) = gsin\theta \qquad \rightarrow \frac{dv}{dt} = gsin\theta - \frac{b}{m}v(t)$$

$$\frac{v_{i+1} - v_i}{t_{i+1} - t_i} = gsin\theta - \frac{b}{m}v_i$$

$$v_{i+1} = \left(gsin\theta - \frac{b}{m}v_i\right)(t_{i+1} - t_i) + v_i$$

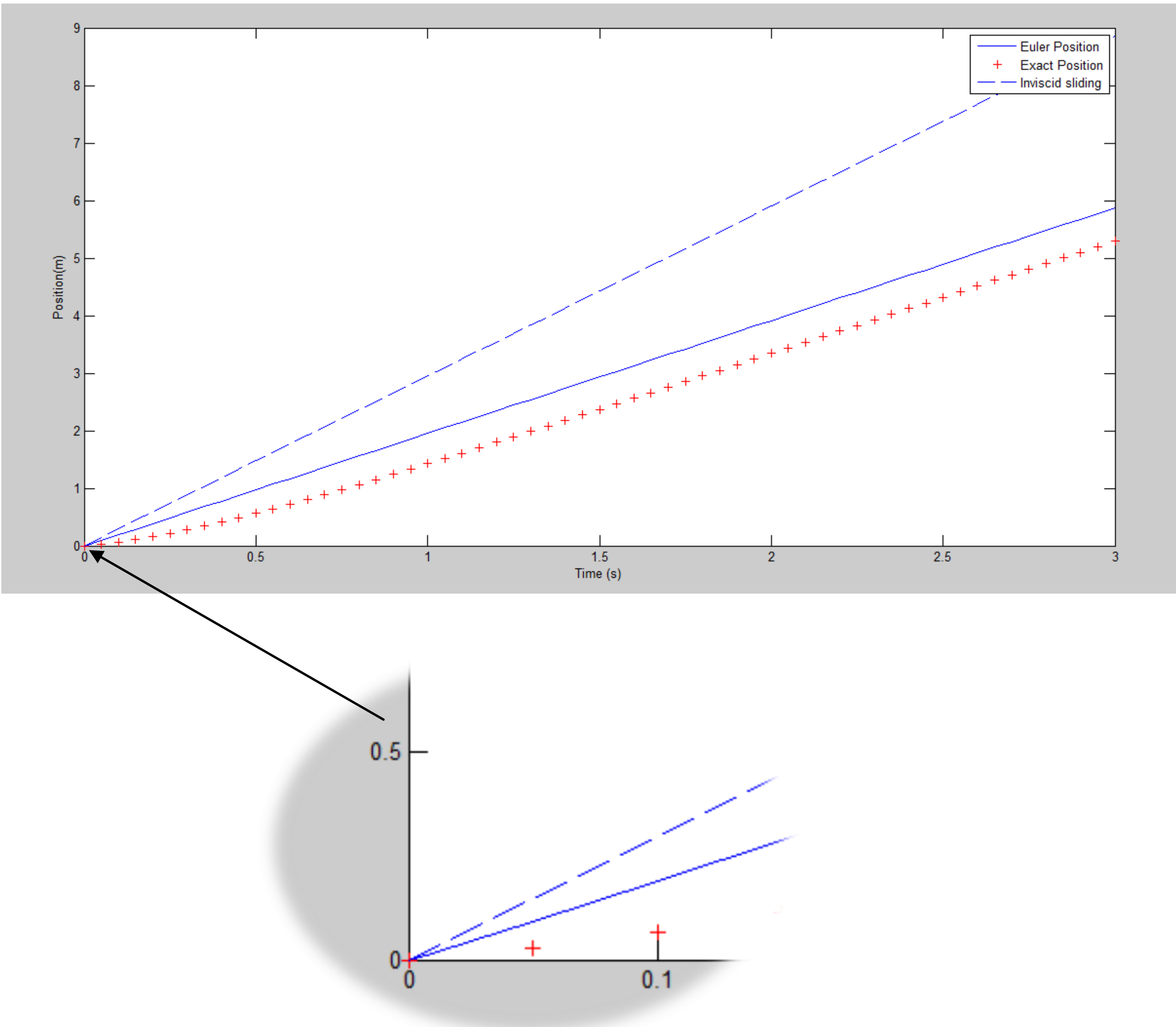


Euler's method for position:

$$v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$\frac{dx}{dt} = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$x_{i+1} = \left(\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right) e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}\right) * (t_{i+1} - t_i) + x_i$$



9. Numerical Solution for the transcendental :

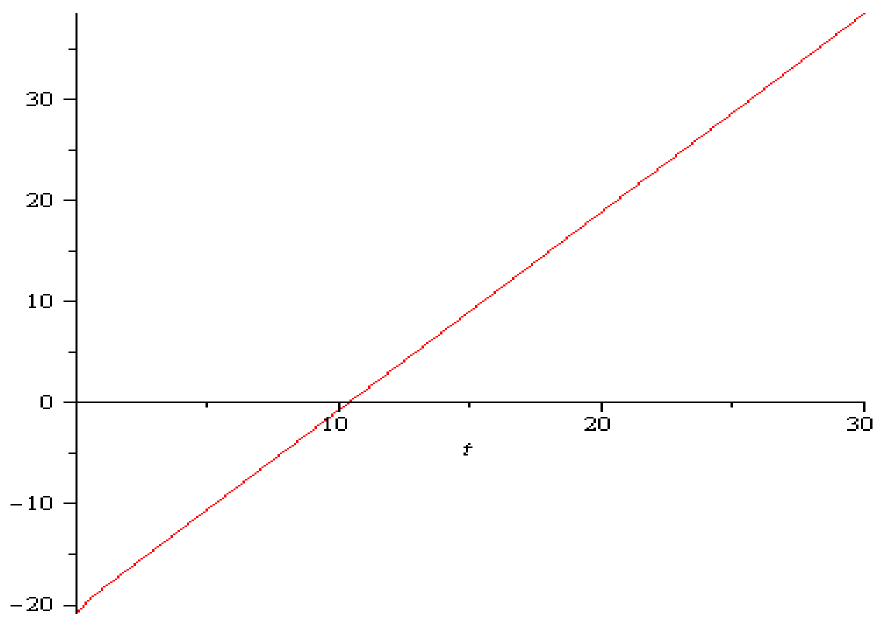
Bisection method to solve for the time (t_b) for viscid case:

$$\frac{H}{sin\theta} = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + \left(\frac{I}{b} - \frac{m^2 gsin\theta}{b^2}\right)$$

> $$y := -\frac{10}{\sin\left(\frac{\pi}{6}\right)} + \frac{2}{5}\left(\frac{1}{2} - 2\cdot 9.8\cdot\frac{\sin\left(\frac{\pi}{6}\right)}{5}\right)\cdot\exp\left(\frac{(-5\cdot t)}{2}\right) + 2\cdot 9.8\cdot\frac{\sin\left(\frac{\pi}{6}\right)\cdot t}{5} + \left(\frac{1}{2} - 4\cdot 9.8\cdot\frac{\sin\left(\frac{\pi}{6}\right)}{25}\right)$$

$$-20.28400000 - 0.5840000000 e^{-\frac{5}{2}t} + 1.960000000 t$$

> $plot(y, t = 0..30)$



Graphical approximation: 10.33 sec

Set the boundary for upper and lower limit:

tl=10

tu=11

Command Window

```
Enter lower limit of approximated root value:  10
Enter upper limit of approximated root value:  11
The root is 10.5021
>>
```

$$v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$v(t_b) = \left(\frac{1}{2} - \frac{2 * 9.8 * \sin{(30)}}{5}\right)e^{\frac{-5t_b}{2}} + \frac{2 * 9.8 * \sin{(30)}}{5}$$

From viscid case we get t_b =10.502 s

$$v(t_b) = \left(0.5 - \frac{2 * 9.8 * \sin{(30)}}{5}\right)e^{\frac{-5t_b}{2}} + \frac{2 * 9.8 * \sin{(30)}}{5}$$

$v(t_{b_{viscid}}) = 1.96\ m/s$

Case 2: *Time to reach the bottom of the slide for inviscid case*, $b = 0$,

$v(t) = gsin\theta t + \frac{I}{m}@t \rightarrow t_b = 2.75$----from the roots in eqn (viii)

$v(t) = 9.8 * \sin(30) * t_b + 0.5$

$v(t_{b_{inviscid}}) = 14\ m/s$

10. Numerical Solution for the potential energy, kinetic energy and dissipated heat:

[Reminder: all symbolic calculation maple are done under impulse, I, represented by, J]

$$Potential\ Energy, P(t) = m * g(H - x(t) * sin\theta)$$
$$= 19.6(10 - x(t) * 0.5)$$
$$= 196 - 9.8x(t)$$

$$Kinetic\ \ Energy, K(t) = 0.5 * m(v(t))^2$$
$$= (v(t))^2$$

$$Total\ heat\ dissipated, Q(t) = \int_0^{t_b} b * (v(t))^2\,dt$$
$$= 5\int_0^{t_b} (v(t))^2\,dt$$

$$heat\ dissipated, D(t) = b * (v(t))^2$$
$$= 5(v(t))^2$$

> $x := -\frac{m}{b}\left(\frac{J}{m} - \frac{(m \cdot g \cdot \sin(\Theta))}{b}\right) \cdot \exp\left(\frac{(-b \cdot t)}{m}\right) + \frac{(m \cdot g \cdot \sin(\Theta))}{b} t + \left(\frac{J}{m} - \frac{(m^2 \cdot g \cdot \sin(\Theta))}{b^2}\right)$

$$0.5840000000 e^{-\frac{5}{2} t} + 1.960000000 t - 0.284000000$$

$v := \left(\frac{J}{m} - \frac{(m \cdot g \cdot \sin(\Theta))}{b}\right) \cdot \exp\left(\frac{(-b \cdot t)}{m}\right) + \frac{(m \cdot g \cdot \sin(\Theta))}{b}$

$$-1.460000000 e^{-\frac{5}{2} t} + 1.96000000$$

$p := 196 - 9.8\,x$ *Potential Energy(p)*

$$198.7832000 - 5.723200000 e^{-\frac{5}{2} t} - 19.20800000 t$$

$k := 0.5 \cdot m \cdot v^2$ *Kinectic Energy(K)*

$$1.0 \left(-1.460000000 e^{-\frac{5}{2} t} + 1.960000000\right)^2$$

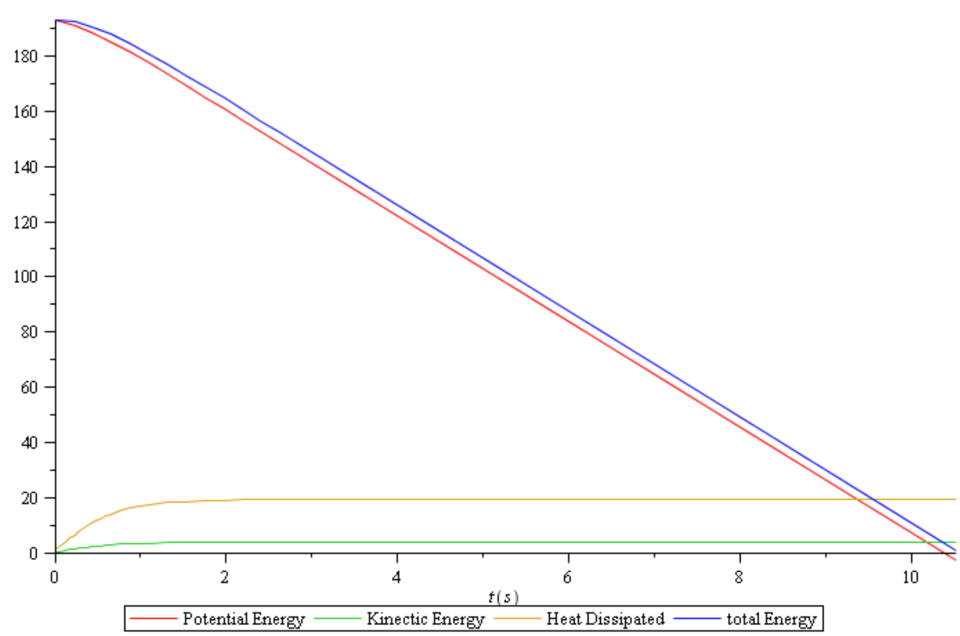
$d := b \cdot v^2$ *heat dissipated(d)*

$$5 \left(-1.460000000 e^{-\frac{5}{2} t} + 1.960000000\right)^2$$

$R := p + k;$ *Total energy(R) = PE + KE*

$$198.7832000 - 5.723200000 e^{-\frac{5}{2} t} - 19.20800000 t + 1.0 \left(-1.460000000 e^{-\frac{5}{2} t} + 1.960000000\right)^2$$

plot([*p*, *k*, *d*, *R*], *t* = 0 ..10.5021)

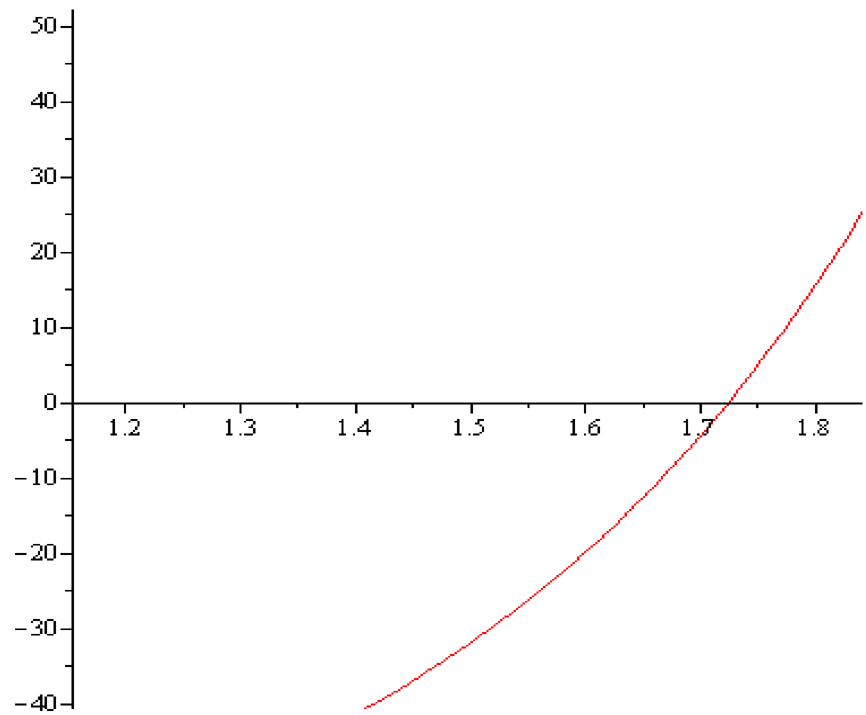


$\int_0^{10.5021} d \, dt$

The total heat dissipated due to damping(Q) = 192.4095368

11. Numerical Solution for the terminal condition:

> $plot(\exp(2.5 \cdot t) - 74.4897, t = 1..2)$




```
Command Window
Enter lower limit of approximated root value:  1
Enter upper limit of approximated root value:  2
The root is: 1.7234
>>
```

$$v(t) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$1.01 * v(\infty) = \left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta}{b}$$

$$1.01 * 1.96 = (-1.46)e^{-2.5t} + 1.96$$

$$e^{2.5t} - 74.4897 = 0$$

$t_{101\%} = 1.7234\ s\ [\ time\ body\ to\ reach\ 101\%\ terminal\ speed]$

$$x(t) = -\frac{m}{b}\left(\frac{I}{m} - \frac{mgsin\theta}{b}\right)e^{\frac{-bt}{m}} + \frac{mgsin\theta * t}{b} + \left(\frac{I}{b} - \frac{m^2gsin\theta}{b^2}\right)$$

$$x(t_{101}) = -0.4 * (-1.46) * 0.01345 + 1.96 * 1.7324 + (0.2 - 0.784)$$

$$x(t_{101}) = -0.4 * (-1.46) * 0.01345 + 1.96 * 1.7324 + (0.2 - 0.784)$$

$$x(t_{101}) = 2.819\ m[\ distance\ the\ body\ travels\ by\ the\ time\ it\ reach\ 101\%\ terminal\ speed]$$

Appendix:

1. MatLab code for Euler and Exact solution for Velocity

```
clear all
clc
% The problem to be solved is:
           %v'(t)+(I/m)v(t)=gsin(theta)

%This problem has a known exact solution
           %v(t)=(I/m-(m*g*sin(theta))/b)*exp((-b*t)/m)+(m*g*sin(theta))/b

% Numerical Parameters
m = 2; % mass of the sliding box [kg]
b = 5; % damping force [N-s/m]
g = 9.8; % acceleration due to gravity [m/s^2]
H= 10; % height of ramp [m]
theta= (30*pi)/180; % slope of the ramp [rad]
I= 1;% impulse of the ramp [degreee]
y_0=0; % initial condition for position [m]
v_0=I/m; % initial condition for velocity [m/s]
v(1)=v_0;
h=0.05;
t_0=0;
t_f=3;
t=t_0:h:t_f;

for i=1:length(t)-1,
    dvdt=(g*sin(theta)-(b*v(i))/m);
    v(i+1)=v(i)+dvdt*h;
end
    V_Exact=(I/m  -(m*g*sin(theta))/b)*exp((-b*t)/m)+(m*g*sin(theta))/b;
    V_inviscid= g*sin(theta)*t+I/m;

plot(t,v)
hold on
plot(t,V_Exact,'r+')
hold on
plot(t,V_inviscid,'b--')
hold off
xlabel('Time (s)')
ylabel('Velocity (m/s)')
legend('Euler Velocity','Exact Velocity','Inviscid sliding')

% function y = ftrans(t)
% y = y =-H/sin(theta)-m/b *(I/m -(m*g*sin(theta))/b)*exp((-
b*t)/m)+(m*g*sin(theta)*t)/b + I/b  -(m^2*g*sin(theta))/b^2;
```

2. MatLab code for Euler and Exact solution for position:

```
clear all
clc
% The problem to be solved is:
           %x'(t)=(I/m  -mgsin?/b)*e^((-bt)/m)+ mgsin?/b

%This problem has a known exact solution
           %x(t)=-m/b*(I/m  -(mgsin(theta))/b)*e^((-bt)/m)+
           %(mgsin(theta))*t)/b+ (I/b -(m^2 gsin(theta))/b^2 )

% Numerical Parameters
m = 2; % mass of the sliding box [kg]
b = 5; % damping force [N-s/m]
g = 9.8; % acceleration due to gravity [m/s^2]
H= 10; % height of ramp [m]
theta= (30*pi)/180; % slope of the ramp [rad]
I= 1;% impulse of the ramp [degree]
x_0=0; % initial condition for position [m]
v_0=I/m; % initial condition for velocity [m/s]
x(1)=x_0; % initializing condition for velocity [m/s]
h=0.05; % step of iteration
t_0=0; % initial time [s]
t_f=3; % final time [s]
t=t_0:h:t_f; % initializing matrix of independent time variable

for i=1:length(t)-1,
    dxdt=(I/m  -(m*g*sin(theta))/b)*exp((-b*i)/m)+((m*g*sin(theta))/b);
    x(i+1)=x(i)+ dxdt*h;
end
    x_Exact=(-m/b)*(I/m  -(m*g*sin(theta))/b)*exp((-b*t)/m)+((m*g*sin(theta)*t)/b)+
(I/b -(m^2*g*sin(theta))/b^2 );
    x_inviscid = (0.5*g*sin(theta))*t+(I/m)*t;

plot(t,x)
hold on
plot(t,x_Exact,'r+')
hold on
plot(t,x_inviscid,'b--')
hold off
xlabel('Time (s)')
ylabel('Position(m)')
legend('Euler Position','Exact Position','Inviscid sliding')
```

3. Numerical solution for transcendental equation:

```
function y = ftrans(t)
% function f1
m = 2; % mass of the sliding box [kg]
b = 5; % damping force [N-s/m]
g = 9.8; % acceleration due to gravity [m/s^2]
H= 10; % height of ramp [m]
theta= (30*pi)/180; % slope of the ramp [rad]
I= 1;% impulse of the ramp [degreee]
y =-H/sin(theta)-m/b *(I/m  -(m*g*sin(theta))/b)*exp((-b*t)/m)+(m*g*sin(theta)*t)/b
+ I/b  -(m^2*g*sin(theta))/b^2;



% function y = ftrans(t)
% y = y =-H/sin(theta)-m/b *(I/m -(m*g*sin(theta))/b)*exp((-
b*t)/m)+(m*g*sin(theta)*t)/b + I/b  -(m^2*g*sin(theta))/b^2;

clear all
clc

tl = input ('Enter lower limit of approximated root value:  ');
tu = input ('Enter upper limit of approximated root value:  ');
tr = (tl + tu)/2;

while abs(ftrans(tr))>0.01
    test = ftrans(tu) * ftrans(tr);  % form test product
    if test < 0
        tl = tr;         % root is above average
    else
        tu = tr;         % root is below average
    end
   tr = (tl + tu)/2;
end
fprintf('The root is %g\n', tr)
```